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SUPPRESSION OF TURBULENCE IN THE CORES OF CONCENTRATED VORTICES

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1. The problem of the motion of vortex rings has intrigued researchers for more than a century now [1]. On the initiative of M. A. Lavrent'ev, the Institute of Hydrodynamics of the Siberian Branch of the Academy of Sciences of the USSR has been conducting experimental and theoretical studies for several years on this effect and other rotational flows of liquids and gases [2]. A mathematical model for the description of the motion of turbulent vortex rings has been proposed on the basis of an analysis of the experimental facts [3, 4]. This model rests on the hypothesis that the turbulent nature of the motion and the transport of a tracer impurity by it can be described by means of scalar coefficients of turbulent viscosity ν and turbulent diffusion $\varkappa$ that vary with time but do not depend on the space coordinates. The additional assumption of flow self-similarity, which is highly consistent with the experimental results, has made it possible to calculate the structure of a vortex ring in the vanishing-viscosity limit [5]; the theory in this case does not contain any empirical constants. However, a comparison of the calculations with the existing experimental results discloses a significant discrepancy in the vicinity of the core of the vortex ring.

It is now clear that the principal cause of this discrepancy is the assumption of turbulence "uniformity" throughout the vortex volume. The results of qualitative experiments and certain theoretical considerations [6] indicate that the core of the vortex ring is almost completely devoid of turbulent tracer transport (the "laminar core" effect) in connection with strong turbulent tracer transport in the atmosphere of the vortex ring. This turbulence suppression is caused by the presence in rapidly rotating flow of a singular "elasticity" associated with the gyroscopic behavior of the fluid particles.

The unsteadiness of the flow in the vortex makes it exceedingly difficult to study the effect discovered in [6]. In this article we describe experiments by which it is possible to observe a similar effect under steady-state conditions; we also present a qualitative explanation of the effect and propose simple models of the turbulent stresses and tracer transport in the cores of line and ring vortices.

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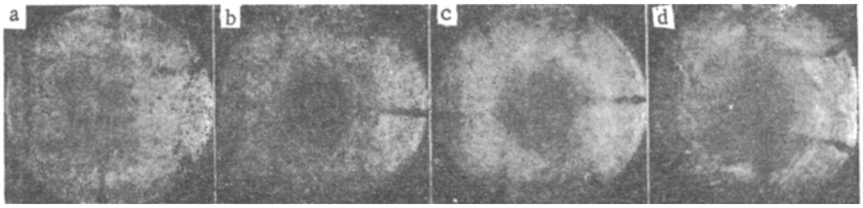

Fig. 1

2. We give the results of experiments that permit observation of the turbulence suppression effect under steady-state conditions. We investigate Couette-type flow between cylinders, the inner one in the form of a grid structure. The experiment entails observing the penetration of turbulence from the space between cylinders ("gap") into a fluid rotating in the inner cylinder. The outer (solid) cylinder in this case is the lateral wall of a water-filled container with a diameter of 17 cm. The grid structure of the inner cylinder is assembled from vertical rods, the number of cross sections of which is varied between definite limits. The outer and inner cylinders can be rotated independently with angular velocities Ω_2 and Ω_1. The experiments show that the central part of the flow contains a rigidly rotating (with velocity Ω) zone. Typical top-view photographs of the flow are shown in Fig. 1. The flow is visualized by means of aluminum powder suspended in the fluid and a slit light source for observation of the horizontal cross section of the flow. This visualization technique works on the directional scattering of light by powder "flakes" oriented under the action of the velocity gradients [7, 8]. The photographs clearly reveal the "laminar core" around the flow-rotation axis and the turbulence "penetrating" that core from the gap. The quantities Ω_1, Ω_2, Ω have the following values (rps): a) 0.22, −1.05, 0; b) 0.4, −0.88, 0.26; c) 0.89, −0.38, 0.83; d) 1.34, 0, 1.33. In order to maintain a roughly equal turbulence strength in the gap the difference $\Omega_1 - \Omega_2$ is made approximately the same in every case. The quantity Ω increases monotonically from Fig. 1a through Fig. 1d. In Fig. 1a, $\Omega = 0$, and the entire flow is turbulent (no "suppression"). With an increase in Ω the radius of the "laminar core" increases, corresponding to ever-increasing "suppression." Hence we infer that the degree of "suppression" is proportional to the mean vortex size. This experimental result can be corroborated by the following technique. We consider the flow corresponding to Fig. 1a, in which turbulence is not suppressed. If now we increase Ω_1 without changing Ω_2, the turbulence strength in the gap increases. However, a velocity $\Omega \neq 0$ develops simultaneously, and a clearly visible "laminar core" appears. Thus, the emerging rotation "suppresses" even the strengthened turbulence in the gap.

We have also visualized the investigated flow by means of a passive dye tracer. After the latter is injected into the gap between the solid wall and the grid, the flow is observed to become rapidly colored everywhere except in the "laminar core" (Fig. 2). In the regime corresponding to Fig. 1a the entire fluid is rapidly colored. Direct measurements of the turbulent tracer-transport velocity yield diffusivity estimates from 1 to 10 cm^2/sec. Turbulent tracer transport is not observed in the "laminar cores."

These qualitative results do not depend on whether or not the inner cylinder has a bottom that rotates with it. The induced secondary flows are very weak. We call attention to three more facts. First, the interfaces between the turbulent and nonturbulent fluids in Fig. 1 are steady. Fluid is not entrained into turbulent motion across these interfaces, or the process of "laminarization" of the turbulent fluid takes place with equal velocity. Second, this experiment evinces the fact that even moderately small Rossby numbers are sufficient for the suppression of turbulence by rotation [8, 6]. Third, the experiments can be regarded as a crude laboratory model of turbulence in the central zones of atmospheric vortices (as in the center of a tornado or the "eye" of a hurricane, etc.).

3. The experiments described above for observing turbulence-suppression effects, like their counterparts for vortex rings, can be explained in terms of the "elasticity" properties of rotational flows and the wave nature of the fluctuating motion [6, 9]. Such qualitative notions provide a certain basis for modeling of the turbulent stresses and tracer transport in flows with concentrated vortices.

We consider a turbulent rotationally symmetric flow with circular mean-flow streamlines. This flow situation represents a first approximation of flows in the vicinity of the "laminar core." For definiteness we specify flow near the core of a vortex ring, deferring the flow self-similarity problem for now.

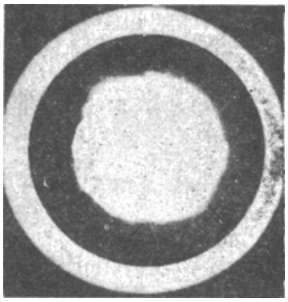

Fig. 2

We adopt the hypotheses of the existence of coefficients of turbulent viscosity $\nu = \nu(r, t)$ and turbulent diffusion $\varkappa = \varkappa(r, t)$

$$\langle uv\rangle = \nu(\partial U/\partial r - U/r),\ \langle cv\rangle = \varkappa\partial C/\partial r,$$

where the angle brackets denote the ensemble average, U, u, C, and c are the mean and fluctuating fields of the angular velocity component and the concentration, and v is the fluctuation component of the radial velocity. The equations for the mean fields U(r, t), $\Omega \equiv \partial U/\partial r + U/r$, and C(r, t) have the form

$$\partial U/\partial t = \nu\partial\Omega/\partial r + \partial\nu/\partial r(\partial U/\partial r - U/r),$$
$$\partial C/\partial t = \partial/\partial r\cdot(\varkappa\partial C/\partial r).$$

For the formulation of models in this scheme it is sufficient to indicate the methods of determination of the functions $\nu(r, t)$ and $\varkappa(r, t)$. We denote by r_0 the radial dimension of the "laminar core." According to the experimental results of [6], the function $\varkappa$ has a more or less sharp jump at r_0. In the "laminar core" the tracer diffusion is molecular, $\varkappa = \varkappa_m$, while outside the core all that can be said is that $\varkappa \gg \varkappa_m$. Following the basic ideas of the model in [4, 3], we assume that outside the core $\varkappa$ settles rapidly into a constant value $\varkappa_0$. The simplest relation of this kind is a step function:

$$\varkappa(r, t) = \begin{cases}\varkappa_m & \text{for} \quad r < r_0,\\ \varkappa_0 & \text{for} \quad r > r_0.\end{cases}$$

In accordance with the general concepts of turbulent transport mechanisms it is reasonable to expect that the function $\nu(r, t)$ will behave similarly:

$$\nu(r, t) = \begin{cases}\nu_1 & \text{for} \quad r < r_0,\\ \nu_0 & \text{for} \quad r > r_0.\end{cases}$$

The quantity ν_1 cannot a priori be set equal to the molecular viscosity, because of the possibility of momentum transport by the wave motion [10, 11]. On the basis of general considerations $\nu_1 < \nu_0$. These expressions correspond to the simple physical notion that for $r > r_0$ rotation does not affect the turbulence characteristics and for $r < r_0$ its influence is so strong that fluctuation motion exists only in the form of "weak" turbulence. Without going into the method of determination of r_0 and other details of the models formulated here, we note that numerical calculations give satisfactory results for the tracer transport process and unsatisfactory results for the Reynolds stresses. The latter fact is attributable to the strong inception of the circulation-excess effect [12], i.e., large negative values of the vorticity. This kind of flow is unstable.

The model described below comprises a method of smoothing the step functions on the basis of the concepts of flow "elasticity" [6, 9]. Such concepts have been used extensively for some time now to model turbulent stresses in stratified as well as in curvilinear flows [10, 13-19].

It is customary in papers on these problems to speak of the stabilizing or destabilizing action of stratification or mean curvature, characterizing these attributes by quantities taken from linear inviscid problems. For example, it has been assumed [10, 18] that the turbulent viscosity coefficient in plane-parallel stratified flows is a single-valued function of the Richardson number Ri. The form of this function is determined from the conditions of matching with experiments and elementary physical considerations. A direct analog of the number Ri in circular flows is the quantity [9]

$$J = \frac{2k^2U\Omega}{r}\Big/\left[m\left(\frac{U}{r}\right)'\right]^2.$$

Here k and m are the wave numbers of the disturbance in the axial and angular directions. A direct translation of the results of [10, 18] to the case of circular flows gives

$$\nu = \nu_0(1+k_1J)^{-\mu_1}, \quad \varkappa = \varkappa_0(1+k_2J)^{-\mu_2} \tag{3.1}$$

with positive constants ν_0, $\varkappa_0$, k_1, k_2, μ_1, μ_2. It follows from physical considerations that tracer transport is more strongly "suppressed" than momentum transport, so that $\mu_2 > \mu_1$. This characteristic is related to the possibility of a "wave" mechanism of momentum transport, whereas the tracer cannot be transported by wave motion [10, 11]. Another possibility is the analogous approximation for the mixing length

$$l = l_0(1+k_3J)^{-\mu_3} \tag{3.2}$$

with its subsequent use for modeling of $\nu(r, t)$:

$$\nu = l^2|\partial U/\partial r - U/r|. \tag{3.3}$$

Outside the core of vorticity J(r) decays rapidly, so that ν_0, $\varkappa_0$, l_0 yield the values of the transport coefficient in the atmosphere of the ring. The ratio k/m entering into J has the significance of the ratio of the turbulence space scales in different directions. The choice of a particular value for this ratio is inconsequential, because the constants k_1, k_2, k_3 are determined by matching with experiment.

We could continue the list of possible approximations and detailing of their argumentation. However, this activity is practical only when adequate experimental evidence is available. At the present time there is only one paper [20] reporting measurements of the time dependence of the averaged parameters of a vortex ring, while measurements of the turbulence characteristics are totally lacking. We therefore limit our discussion to the observation that numerical calculations indicate the possibility of matching with the data of [20] within the context of any one of the models (3.1)-(3.3).

4. We conclude with a formulation, based on the experimental facts and considerations set forth above, of the problem of self-similar turbulent vortex rings in the vanishing-viscosity limit [5] with regard for the dependence of the turbulent viscosity coefficient on the coordinates. We proceed from the assumption that the Reynolds stresses are representable in the form

$$\langle u_i u_k\rangle = \frac{1}{2}\langle u_l^2\rangle\delta_{ik} - \nu_*(t, \mathbf{r})\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right).$$

In a cylindrical coordinate system with axial symmetry, neglecting molecular viscosity, we obtain the following equation for the azimuthal component of the vorticity vector Ω_φ for the averaged flow:

$$\begin{aligned}\frac{\partial\Omega_\varphi}{\partial t} + \frac{\partial\Psi}{\partial r}\frac{\partial}{\partial z}\left(\frac{\Omega_\varphi}{r}\right) - \frac{\partial\Psi}{\partial z}\frac{\partial}{\partial r}\left(\frac{\Omega_\varphi}{r}\right) = \frac{\partial}{\partial z}\left(\nu\frac{\partial\Omega_\varphi}{\partial z}\right) + \frac{\partial}{\partial r}\left(\nu\frac{\partial\Omega_\varphi}{\partial r}\right)\\ + \frac{\partial}{\partial z}\left(d_{rz}\frac{\partial\nu}{\partial z} - d_{zz}\frac{\partial\nu}{\partial r} + \nu\frac{d_{rr}}{r}\right) - \frac{\partial}{\partial r}\left(d_{rz}\frac{\partial\nu}{\partial r} - d_{rr}\frac{\partial\nu}{\partial z} + \nu\frac{d_{zr}}{r}\right),\end{aligned} \tag{4.1}$$

where Ψ is the stream function, $v_z = (1/r)(\partial\Psi/\partial r)$ and $v_r = -(1/r)(\partial\Psi/\partial z)$ are the axial and radial velocity components, and d_{zz}, d_{rz}, d_{rr} are the corresponding components (multiplied by two) of the strain-rate tensor:

$$d_{zz} = 2\frac{\partial v_z}{\partial z}; \quad d_{zr} = \frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}; \quad d_{rr}' = 2\frac{\partial v_r}{\partial r}.$$

Considering the motion to be self-similar with governing parameter P_0 ($[P_0] = L^4/T$) [3, 4], we obtain

$$\Omega_\varphi = \frac{1}{t}\bar{\omega}(x, y), \quad \Psi = \frac{P_0^{3/4}}{t^{1/4}}\bar{\psi}(x, y), \quad x = \frac{z}{P_0^{1/4}t^{1/4}}, \quad y = \frac{r}{P_0^{1/4}t^{1/4}}.$$

The time dependence of the turbulent viscosity coefficient is determined by dimensional analysis and has the form

$$\nu_*(t, \mathbf{r}) = \lambda_0\frac{P_0^{1/2}}{t^{1/2}}\lambda(x, y),$$

where λ_0 is a constant and $\lambda(x, y) \leq 1$. The quantity λ_0 is small. It is reasonable, therefore, to consider the limiting case $\lambda_0 \to 0$. We make a change of variables, putting

$$\xi = \frac{1}{\lambda_0^{1/2}}\left(x - \frac{1}{\lambda_0^{3/2}}\xi_0\right), \quad \eta = \frac{1}{\lambda_0^{1/2}}y, \quad \mu = \frac{1}{\lambda_0^2},$$
$$\bar{\omega} = \frac{1}{\lambda_0^2}\,\omega(\xi, \eta), \quad \bar{\psi} = \frac{1}{\lambda_0^{1/2}}\left(\psi + \frac{1}{8}\xi_0\eta^2\right).$$

This change corresponds not only to elongation, but also to transformation to a coordinate system attached to the vortex ring. The quantity ξ_0 is determined by the requirement that the maximum of ω lies on the line $\xi = 0$.

In these variables Eq. (4.1) takes the form

$$\frac{\partial}{\partial\xi}\left(\lambda\frac{\partial\omega}{\partial\xi}\right) + \frac{\partial}{\partial\eta}\left(\lambda\frac{\partial\omega}{\partial\eta}\right) + \frac{\partial}{\partial\xi}\left(d_{\xi\eta}\frac{\partial\lambda}{\partial\xi} - d_{\xi\xi}\frac{\partial\lambda}{\partial\eta} + \lambda\frac{d_{\eta\eta}}{\eta}\right)$$
$$- \frac{\partial}{\partial\eta}\left(d_{\xi\eta}\frac{\partial\lambda}{\partial\eta} - d_{\eta\eta}\frac{\partial\lambda}{\partial\xi} + \lambda\frac{d_{\xi\eta}}{\eta}\right) + \frac{1}{4}\xi\omega_\xi + \frac{1}{4}\eta\omega_\eta + \omega = \mu\left[\frac{\partial\psi}{\partial\eta}\frac{\partial}{\partial\xi}\left(\frac{\omega}{\eta}\right) - \frac{\partial\psi}{\partial\xi}\frac{\partial}{\partial\eta}\left(\frac{\omega}{\eta}\right)\right]. \qquad (4.2)$$

The boundary conditions required for solving this equation coincide with the conditions in [5].

In this coordinate system the lines of constant $\Omega = \omega/\eta$ almost coincide with the lines ψ = const and are close to circles near the maximum of ω for large values of μ. It is reasonable to assume, therefore, that the dependence of $\lambda(\xi, \eta)$ on the coordinates will be well described by one of the above-considered approximations of the turbulent viscosity coefficient for flows with circular streamlines.

The following quantity can serve as an analog of the Richardson number for flow in the vicinity of the core of a vortex ring:

$$I = \frac{2U\Omega_\varphi}{R\,|\,d_{\tau n}|^2},$$

where $U = \sqrt{v_r^2 + v_z^2}$; R is the radius of curvature of the streamline passing through a given point (in a coordinate system moving with the vortex); and $d_{\tau n}$ is twice the component of the strain-rate tensor (τ is the tangent vector, and $\mathbf{n}$ is the normal to the streamline). It can be shown that

$$d_{\tau n} = \Omega_\varphi - 2U/R.$$

Considering the qualitative character of the arguments used in selecting the form of the turbulent viscosity coefficient as a function of the coordinates, it seems logical to choose the simplest function. Accordingly, we assume that ν_* is constant on the isolines $\Omega = \omega/\eta$ = const and

$$\lambda(\xi, \eta) = \lambda(\Omega) = [1 + \beta I(\Omega)]^{-k}, \qquad (4.3)$$

where $I(\Omega)$ is given by the equations

$$I(\Omega) = \frac{\Omega\bar{\Omega}}{(\Omega - \bar{\Omega})^2}, \quad \bar{\Omega} = \frac{\Gamma(\Omega)}{S(\Omega)},$$
$$\Gamma(\Omega) = \int\int \eta\Omega\, d\xi\, d\eta, \quad S(\Omega) = \int\int \eta\, d\xi\, d\eta.$$

Here the integration is carried out over the domain delineated by the closed line Ω = const.

Integrating (4.2) over a domain with boundary delineated by a certain closed streamline, for any μ we obtain

$$\oint \lambda(\Omega)\,\eta\nabla\Omega\cdot\mathbf{n}\,dl + \oint\frac{\lambda}{\eta}(d_{\eta\eta}n_\xi - d_{\xi\eta}n_\eta)\,dl + \oint\left[\left(d_{\xi\eta}\frac{\partial\lambda}{\partial\xi} - d_{\xi\xi}\frac{\partial\lambda}{\partial\eta}\right)n_\xi - \left(d_{\xi\eta}\frac{\partial\lambda}{\partial\eta} - d_{\eta\eta}\frac{\partial\lambda}{\partial\xi}\right)n_\eta\right]dl$$
$$+ \frac{1}{4}\oint \xi\eta\Omega n_\xi\,dl + \frac{1}{4}\oint \eta^2\Omega n_\eta\,dl + \frac{1}{2}\int\int \eta\Omega\,d\xi\,d\eta = 0,$$

where dl is an element of length of the streamline and $\mathbf{n}$ is the unit normal to the streamline. Now, comparing the limiting transition $\mu \to \infty$ and making use of the fact that $\Omega \to \Omega(\psi)$ in that transition, we obtain

$$\lambda(\psi)\,P(\psi)\frac{d\Omega}{d\psi} + \frac{d\lambda}{d\psi}\left(P(\psi)\,\Omega - 2\oint\frac{U^2}{R}\,dl\right) = \frac{1}{2}\Gamma(\psi) + \frac{3}{4}S(\psi)\,\Omega(\psi),$$

where

$$P(\psi) = \int\int \eta^3 \Omega d\xi d\eta; \; S(\psi) = \int\int \eta d\xi d\eta, \; \Gamma(\psi) = \int\int \eta\Omega d\xi d\eta.$$

Here the integration is carried out over the domain bounded by the closed streamline. Thus, in the limit $\mu \to \infty$, as in the constant-viscosity case, for the determination of the structure of the vortex ring we arrive at the problem of matching the potential inviscid flow outside the vortex atmosphere with the rotational inviscid flow inside the atmosphere [5, 2]. At the boundary of the atmosphere the conditions of continuity of ψ and $\Delta\psi$ must hold, with $\Omega(0) = 0$. The variable turbulent viscosity $\lambda(\psi)$ entering into Eq. (4.4) is given by relation (4.3).

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